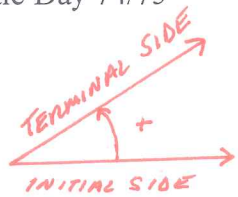
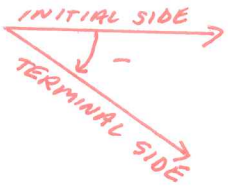
Sec. 7.2 The Sine and Cosine Functions

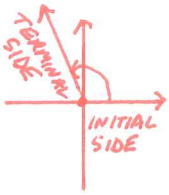


Ray – starts at a point V and extends indefinitely in one direction

Angle – two rays drawn with a common vertex

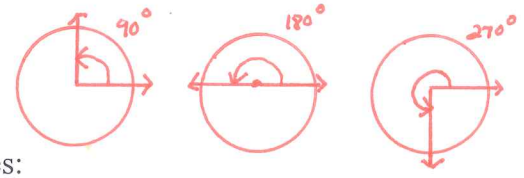
1. **Initial side** – one ray of the angle
2. **Terminal side** – the other ray of the angle
3. **Positive rotation** – happens in counterclockwise rotation
4. **Negative rotation** – happens in clockwise rotation





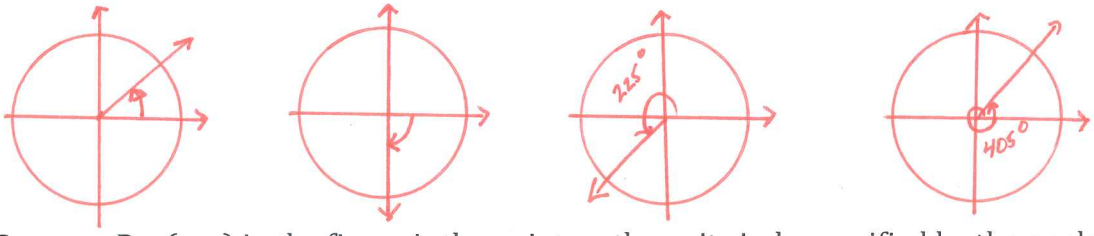
Standard position – if the vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis

Quadrantal Angle – when an angle is in standard position and its terminal side lies on a quadrant (90, 180, or 270)
Right angle – 90 degrees
Straight angle – 180 degrees



Ex. Draw the following angles:

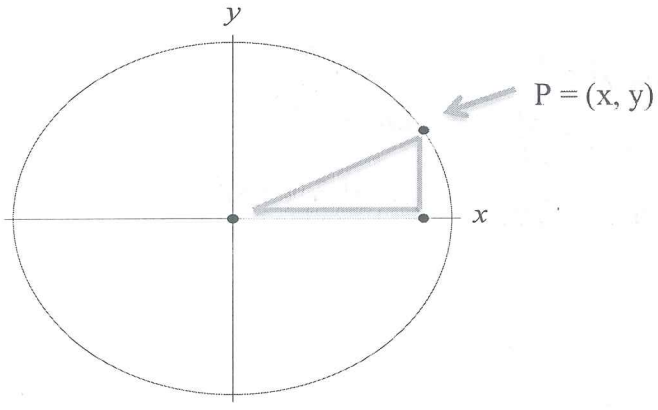
a. 45 degrees b. –90 degrees c. 225 degrees d. 405 degrees



Suppose P = (x, y) in the figure is the point on the unit circle specified by the angle θ. We define the functions, cosine of θ, or cos θ, and sine of θ, or sin θ, by

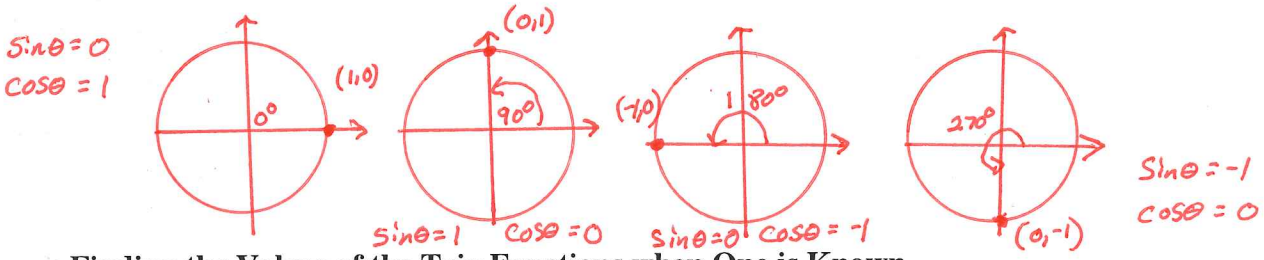
cos θ = *x* and sin θ = *y*.

In other words, cos θ is the *x*-coordinate of the point P; and sin θ is the *y*-coordinate.



Remember that all points in the unit circle are written as (cos θ, sin θ) and $\sin^2\theta + \cos^2\theta = 1$.

Ex. Find the values of sin θ and cos θ for θ = 0°, 90°, 180°, 270°.



Finding the Values of the Trig Functions when One is Known

Method 1:

a. Draw a circle showing the location of the angle θ and the point P = (x, y) that corresponds to θ. The radius of the circle is $r = \sqrt{x^2 + y^2}$.

b. Assign a value to two of the three variables x, y, r based on the value of the given trig function.

c. Use the fact that P lies on the circle $x^2 + y^2 = r^2$ to find the value of the missing variable.

d. Apply any necessary theorems.

Review:

1. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$

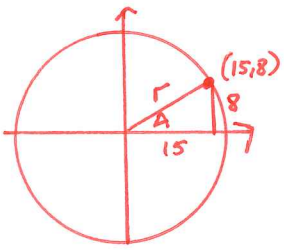
2. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$

Solve the formulas from above for x and y using r for hypotenuse:

$r \cdot \sin\theta = \frac{y}{r} \cdot r$ $\qquad$ $r \cdot \cos\theta = \frac{x}{r} \cdot r$

$r\sin\theta = y$ $\qquad$ $r\cos\theta = x$

Ex. Given the point (15, 8) is on the circle, find the sin A and cos A.



$r^2 = 15^2 + 8^2$

$r = \sqrt{225+64}$

$r = \sqrt{289}$

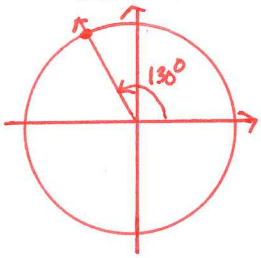
$r = 17$

$\sin A = \frac{o}{h}$ $\qquad$ $\cos A = \frac{a}{h}$

$\sin A = \frac{8}{17}$ $\qquad$ $\cos A = \frac{15}{17}$

$\sin A = .4706$ $\qquad$ $\cos A = .8824$

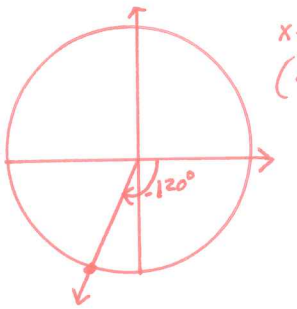
Ex. Given a unit circle and an angle of 130 degrees, find the coordinates of the point that lies on the circle.



$(\cos 130°, \sin 130°)$

$(-.6428, .7660)$

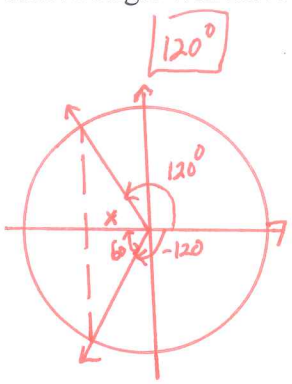
Ex. Give a circle or radius 4 and an angle of -120 degrees, find the coordinates of the point on the circle at that angle. What other positive angle will have the same x-coordinate? Same y-coordinate?

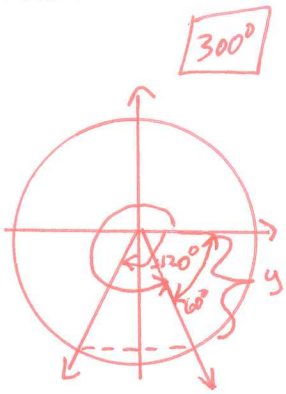


$x = r\cos\theta \quad y = r\sin\theta$

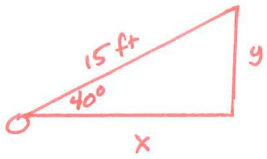
$(4\cos -120°, 4\sin -120)$

$(-2, -3.464)$





Ex. If a shark has a harpoon mounted to its head and the harpoon fires from sea-level at a 40 degree angle and travels 15 feet before it strikes the ship, how far is the shark from the ship? How high up on the ship does the harpoon strike?



$\cos\theta = \frac{a}{h} \qquad \sin\theta = \frac{o}{h}$

$\cos 40° = \frac{x}{15} \qquad \sin 40° = \frac{y}{15}$

$15\cos 40° = x \qquad 15\sin 40 = y$

$11.491 \text{ ft} = x \qquad 9.642 \text{ ft} = y$

HW: pg 282 – 283, #1-4, 6-17, 22-24 **(Four decimal places on 1, 7-10)**